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TWO ALTERNATIVES OF MAGNETIC CUMULATION

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This paper deals with phenomena leading to a considerable increase in magnetic field and energy density during compression of a magnetic flux trapped by a conducting shell and joint deformation of a magnetic field and material. The main features and merits of these two alternative schemes of magnetic cumulation are discussed. A comparison is made between the classical and shock-wave schemes of magnetic compression in a material with a phase transition from a nonconducting to a conducting state. The possibility of magnetic-energy cumulation during stretching of magnetic field line by a transverse flow of a conducting material is considered.

INTRODUCTION

One of the most brilliant scientific achievements of Academician M. A. Lavrent'ev was the development of a model and theory of hydrodynamic cumulation [1]. The phenomenon of hydrodynamic cumulation is unique: in contrast to most natural phenomena leading to smoothing of all nonuniformities in distributions of physical properties, cumulative flows are accompanied by an increase in energy density due to their capability for transferring energy from large to small fluid masses. Precisely in cumulative flows, record energy densities and extreme states of materials have been obtained. It was found that, as a first approximation, most of these phenomena have a tendency toward unlimited cumulation [2].

A fruitful direction of energy cumulation was discovered in 1951 by Academician A. D. Sakharov, who proposed fast compression of a magnetic field by a closed conducting shell [3]. Somewhat later, the same idea was advanced independently by K. M. Fowler from the Los Alamos Laboratory [4]. In the history of magnetic cumulation research over half a century there were periods of total optimism and great success and, perhaps, no less great disappointment. But, owing to the ingenuity and keen intuition of researchers, strikingly high energy densities, several hundred times higher than the energy densities of commercial chemical explosives and lower only than those of nuclear warfare, were achieved exactly in magnetic cumulation experiments [5].

In 1978, we proposed a shock-wave scheme of compression of a magnetic flux together with a material by a closed system of shock waves, which transformed the compressed nonconducting material to a conducting state. Somewhat later, the same idea was proposed independently by K. Nagayama from the University of Kumamoto [8]. In shock-wave compression, in contrast to classical magnetic cumulation, a field is mixed with a material from the very beginning and is compressed together with it, which leads to a number of new physical phenomena. Field and material compression accompanied by an increase in magnetic energy density is possible not only in shock-wave driven magnetic flux compression by a material with rather unusual electric conduction but also in a specially organized joint flow of a field and a conducting material transferring the field. In the following sections, we will discuss the main features of these two alternative schemes of magnetic cumulation.

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1. CLASSICAL MAGNETIC CUMULATION

Physical Conditions of Magnetic Cumulation. The classical method of producing ultrastrong magnetic fields involves compression of a magnetic flux by a closed conducting cylindrical shell that converges to the axis. This shell is usually called a liner. When a liner moves in a magnetic field, a current is induced in a thin layer on the inner surface of the liner. This current maintains and strengthens the magnetic field in the liner cavity, and the current layer is called a skin layer. Interaction of the current with the field gives rise to pressure $p_m = B^2/(8\pi)$ in the liner material, and ohmic heating leads to a rise in temperature. We note an important characteristic feature of magnetic cumulation: when the thickness of the conducting shell is sufficiently large (larger than the thickness of the skin layer), the total magnetic flux, which is equal to the sum of the fluxes in the skin layer and the cumulative cavity, remains unchanged. This is the most general property of MHD flows with conductors having sufficiently large dimensions. In [9], an example of a conducting liquid flow is given where not only is the total flux conserved but, despite the change in the dimensions of the cavity with the field, both the flux in the cavity and the flux in the skin layer remain constant independently.

Using the formulated condition of flux conservation and writing the energy balance equation, it is possible to obtain a physical estimate for the temperature on the skin layer surface. Let us consider diffusion of a homogeneous magnetic field from a cavity with cross section S and perimeter p. At the initial moment, the field B_0 is concentrated only in the cavity, but after a time, diffusing into the conductor to depth $\delta(t)$, the field becomes equal to $B(t) = B_0 S/(S + p\delta(t))$ because of flux conservation. Thus, the magnetic field energy decreases, and the following amount of heat is released in the skin layer per unit length of the cavity along the axis:

$$\Delta Q = \frac{1}{8\pi}\left(B_0^2 S - B^2(t)(S + p\delta(t))\right) = \frac{B_0 B}{8\pi}\, p\delta(t).$$

The increase in temperature in this case is

$$\Delta T = \frac{\Delta Q}{c_V p\delta(t)} = \frac{B_0 B}{8\pi c_V}.$$

Here c_V is the heat capacity per unit volume of the conductor. Setting $c_V = 4$ J/(cm$^3\cdot$ K), we find that in a field of 1 MG, the conductor is heated to about 10^3 K.

The electric fields accompanying magnetic cumulation are insignificant: $E \approx (v/c)B$, i.e., at a compression rate of 1 cm/sec in a field of 1 MG, they are about 1 kV/cm. The squeezing of a magnetic flux from a wedge with opening angle α leads to multiple reflection of the electromagnetic wave from the conducting walls of the wedge, as a result of which the field along the normal to the fixed wall of the wedge increases by a factor of $1/\sin\alpha$ [10].

Constraints. In ideal magnetic cumulation, the flux in the cavity is constant. As a result, a decrease in the cross section of the cavity from S_0 to S leads to an increase in the field $\beta = B/B_0 = S_0/S$ and to the same increase in the magnetic-field energy. For ideal cumulation there is the only constraint — energy, and

$$\beta = B/B_0 = (B^2 S)/(B_0^2 S_0) = U/U_{m0} = \varepsilon + 1. \qquad (1)$$

Here U is the magnetic field energy at the end of compression and ε is the ratio of the kinetic energy of the shell to the initial magnetic energy U_{m0}. Paradoxical as it is, the maximum field for ideal cumulation at the end of compression is inversely proportional to the initial field.

In experiments, the effects of finite conductivity and compressibility of materials are superimposed on the energy constraint, and in extreme fields, a question arises of the possibility of two catastrophes:

1) full loss of conductivity due to overheating of the skin layer by the current and rapid evaporation of the layer;

2) instability of the conductor surface in a strong magnetic field and the associated limitation on the achievable degree of compression.

The dependence of the field amplification on the electric conductivity and features of compression of the cavity can be estimated as follows [11]. Let us introduce the characteristic cross-sectional dimension of the cavity, defining it by the relation $a = 2S/p$, where S is the cross section of the cavity and p is its perimeter. The rate of decrease in magnetic flux in the cavity can be described by the flux relaxation time τ_r. For a cavity with cross-sectional dimension a, the relaxation time can be defined as the time during which the skin-layer thickness reaches a value a, i.e., according to [12], for a material with constant conductivity σ,

$$\tau_r = \frac{4\pi\sigma}{c^2} a^2 = \frac{a^2}{\nu_m}.$$

Here ν_m is the magnetic viscosity. The compression of the cavity is in turn characterized by the compression time τ_c, which depends on the dimension of the cavity a and the kinematics of compression. The efficiency of magnetic flux compression is defined by the ratio of these two times:

$$\mathrm{Re}_m = \tau_r/\tau_c. \tag{2}$$

The quantity Re_m is called the magnetic Reynolds number. It is clear that the larger Re_m, the more efficient the magnetic compression. Because the flux loss is proportional to the perimeter of the cavity and the flux in the cavity is proportional to the cross section, it follows that dissipative processes are growing in importance as the dimensions decrease, and with closure of the cavity, they become predominant. The critical dimension of the cavity can be defined by the condition that the time of relaxation from the cavity equals the compression time $\tau_r(a_*) = \tau_c(a_*)$. Furthermore, it is assumed that when the dimension of the cavity decreases from the initial value a_0 to a_* there is no flux loss, and once the critical dimension is reached, the amplification of the field ceases, i.e., the field amplification is given by

$$\beta_* = S_0/S_* = S(a_0)/S(a_*).$$

It is easy to see that the compression time depends not only on the dimension of the cumulative cavity but also on the kinematics of compression, and the cross section of the cavity is determined by the compression geometry. From the above considerations, it is possible to obtain this dependence for each particular case. A comparison with exact solutions of problems of magnetic cumulation with flux diffusion shows that the above estimates give correct values for the exponent in the dependence of the limiting field amplification on the magnetic Reynolds number and can be used to predict results of experiments with change in scale. The general conclusion is that with increase in the scale of the experiment, the field increases until it reaches a certain critical value that leads to a change in the nature of the flux loss from the cumulative cavity.

The compressibility and heating of the conductor by currents impose fundamental limitations on achievable fields. A strong magnetic field generates a shock wave that propagates into the material and is accompanied by a current zone. In extreme fields, one might expect that the heating of the material is so intense that its conduction and interaction with the magnetic field can be ignored from a certain moment. In this case, nothing prevents the compressed and heated material from expanding to the free surface bordering the magnetic field. A situation known in detonation theory arises: the heat release by the induced currents results in formation of a rarefaction wave behind the shock wave. In this rarefaction wave, the velocity of the material relative to the wave front increases and can be supersonic. Assuming that the heat release per unit volume of the material is equal to the magnetic energy density, ignoring the initial pressure, and performing manipulations that are well known from the elementary theory of detonation, one readily finds that for stationary propagation of a structure consisting of a shock wave, a current layer, and a rarefaction wave, sustained by mechanical and thermal interaction of the conductor with the strong magnetic field, the mass velocity at the Jouguet point should be equal to the Alfén velocity [10] $u_a = B/\sqrt{4\pi\rho}$. Assuming that the conducting material flows with velocity v into the region occupied by the magnetic field, it is easy to see that field compression occurs if $v > u_a$ or

$$\rho v^2/2 > B^2/(8\pi), \tag{3}$$

TABLE 1

B, MG	w_B, kJ/cm^3	v_0, km/sec	w_B/w_D
1.4	8	1.5	1
5.6	128	6	16
14	800	15	100
28	3200	30	400

i.e., as long as the hydrodynamic thrust exceeds the magnetic pressure. From qualitative considerations, this result, as the main limitation of the magnetic field magnitude, was obtained by A. D. Sakharov and given in his review [13].

Conclusions from Physical Estimates. The above estimates are qualitative and show in what direction efforts should be concentrated to obtain record energy densities by the magnetic cumulation method. These estimates lead to the following two conclusions:

1. The scale of experiments should be rather large. This means that it is necessary to produce a strong initial field, make a liner of a dense material, and impart to it the highest possible velocity.

2. Having reached a limit of the field with a particular explosive unit, one might hope to surpass this limit by increasing the dimensions and improving the acceleration of the shell.

To orient ourselves in the required parameters of magnetocumulative (MC) systems, we compiled a table that gives the magnetic field energy density w_B, the required velocity of a material with a density of 8 g/cm^3 according to (3), and the ratio of the magnetic field energy density to the SW energy density $w_D = 8$ kJ/cm^3.

From Table 1 it follows that the production of a field of 1.5 MG is quite realistic and is likely to involve no difficulties in an explosive experiment. However, the value of 14 MG stated by K. M. Fowler in the first publication corresponds to a 100-fold increase in the energy density and requires a liner velocity of 15 km/sec. This result does not appear by itself, and if it suddenly does, this is most likely due to a favorable concurrence of circumstances. Fields exceeding 15 MG are even more problematic.

This proved to be just so. In none of the experiments of the 1960s was it possible to produce fields of 14 and 25 MG stated by Fowler [4] and Sakharov [13], respectively. By the end of the 1970s, the result of magnetic cumulation research was the production of fields of 4–5 MG in poorly reproducible experiments. A number of countries (Great Britain, France, and Italy) ceased studies, and investigations were continued only at the powerful centers on development of nuclear weapons in the USSR and the U.S.A.

Generators of Record Magnetic Fields. The subsequent progress in magnetic cumulation research was due to the following important events:

— the design of a solenoid of an initial field from a composite material whose conductivity is anisotropic and varies under compression, and combination of the functions of an initial field solenoid and a field-compressing liner in a single design;

— direct experimental observations of the mixing of a field with a liner material and measurements of the critical field leading to a catastrophic failure of the current-carrying surface;

— conversion to compression of supercritical fields by a system of cascades, in which, every time mixing of the field and material begins, the conducting material overheated by the current is replaced by the fresh conductor produced by compression of the following cascade.

These outstanding results were obtained by a remarkable team of researchers headed by Academician A. I. Pavlovskii from the Soviet nuclear center at Arzamas-16 and were published in the Proceedings of the IInd, IIIrd, and IVth International Megagauss Conferences [14–16]. The modern state of the art of the problem is considered in detail in a collection of papers dedicated to the 50th anniversary of the All-Russia Research Institute of Experimental Physics [17–19], and in materials submitted to the Commission on Awarding the State Prize of Russia [5].

An initial field solenoid in the generators described was a multilayer (9 layers) and multirun cylindrical

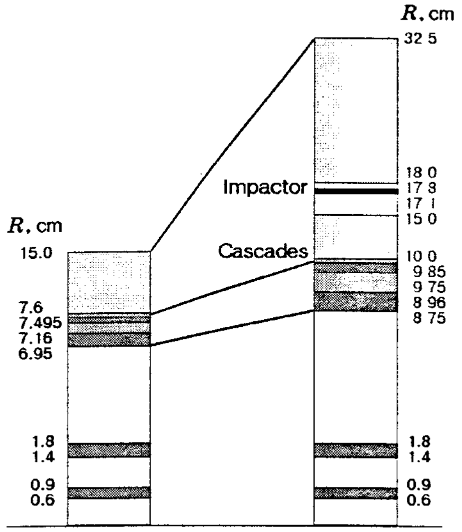

Fig. 1. Diagram of generators for fields of 10 (on the left) and 20 MG (on the right).

structure consisting of a large number (4500) of thin copper wires of 0.25-mm diameter insulated from each other and combined into a single unit by impregnation and sintering in an epoxy compound. After explosion of a cylindrical explosive charge placed above the solenoid, the insulation between the wires failed, and the wires formed a homogeneous liner with a high initial density of 6 g/cm^3 and high conductivity. Such an unusual solenoid could stably produce a strong initial field of 160 kG in a large volume without failure, and after ignition of the explosive charge, it ensured reliable magnetic flux trapping and compression. In experiments with this solenoid, fields of 5–6 MG were achieved but it was not possible to exceed 6 MG. Flash radiography clearly showed that in liner compression without a field, the compression symmetry is preserved fairly well up to complete closure of the cumulative cavity. Experiments with a field showed that once 3.5 MG was reached, intense mixing of the field and the liner material began and then became catastrophic.

The production of fields exceeding 5 MG was achieved with the use of cascades — cylindrical tubes placed in the field compression region at certain distances from the axis of the system and manufactured, as well as the solenoid, from wires insulated from each other. A reliably working three-cascade generator of magnetic fields of 10 MG was designed, and a series of unique experiments was performed to study the behavior of materials in such fields. After a marked increase in the dimensions of the system and considerable improvement of the explosive acceleration system, fields in excess of 20 MG were produced in two experiments, which was reported at the VIIIth Megagauss Conference [20]. The maximum measured field was 28 MG. The energy density of the field was 400 times higher than the energy density of the explosive! This record was achieved by increasing the weight of the explosive charge from 16 to 170 kg and the diameter of the system from 30 to 65 cm. In addition, a two-cascade system for explosive acceleration of the liner was designed to obtain a high compression rate. Figure 1 gives a diagram and the main overall dimensions of two MC-systems for 10 and 20 MG [5].

2. COMPRESSION OF A FIELD WITH A MATERIAL

In classical MC-generators with compression of a magnetic field by a conducting shell, the field and the material are separated: the field is in the generator cavity, the energy necessary for compression is concentrated in the moving shell, and the current flows in a thin layer of the conductor on the boundary with the field. The instability of this boundary leads to mixing of the field with the material and limits the magnitude of attainable fields.

In contrast to classical generators, in new approaches to the production of ultrastrong magnetic fields, a field is mixed with a material from the very beginning. One approach uses nonconducting materials that become conducting under compression. An initial magnetic field can be easily produced in a sample of such a material. Then, compressing this sample with a field by a closed system of shock waves convergent to the axis, it is possible to convert the material to a conducting phase, trap the magnetic flux in the conductor formed, and implement magnetic cumulation by considerably increasing the magnetic field and the energy density in it. This direction of magnetic cumulation was called shock-wave compression of a magnetic field.

However, the possibilities of amplifying fields by joint motion of a field and a material are not limited to shock-wave compression. It is known that in stretching of a magnetic field line, work is also done and the energy of the field also increases. This is a classical proposition of magnetohydrodynamics, plasma physics, and some areas of astrophysics [12], which has found a technical implementation as an original generator of powerful current pulses — the so-called compulsator [21]. However, as applied to ultrastrong magnetic fields, this idea was not considered in detail. This omission was recently pointed out by S. V. Fedorov with collaborators in [22], where, using numerical calculations, they showed the possibility of achieving Megagauss fields by joint deformation of a field and a conductor, accompanied by stretching of magnetic field lines.

Shock-Wave Compression of a Magnetic Field. *Differences and Merits.* The main difference of the shock-wave method of magnetic compression from the liner method is that a conduction wave moves over a material at wave velocity D and the work on field compression is done by material particles moving at mass velocity $u < D$. During compression and conversion of the material to a conducting state, the magnetic field becomes "frozen" in the material and is carried away from the compression region with velocity $(D - u)$ relative to the wave front. At the same time, an extended zone with smooth distributions of magnetic field and current forms behind the wave, and the field diffusion in this zone is insignificant [23].

The convective removal of a large fraction of the magnetic flux severely constrains the energy possibilities of the method. However, in spite of this, compression of a magnetic field with a material has a number of advantages due to generation of a current in a "fresh" conducting material and the possibility of using hydrodynamic cumulation to increase attainable magnetic fields. The shock-wave method provided fields of 3.5 MG with a very good magnetic field amplification factor $\beta = B/B_0 \approx 90$ [23–25].

General energy estimates for a simple model of a compressible medium in which a shock wave transforms the material into an ideal conductor by packing it from the initial density ρ_0 to the final density ρ_f [23], and more detailed calculations [24] showed the fundamental dependence of the potentials of the method on the packing parameter $n = \rho_f/\rho_0 = D/(D - u)$ and led to the following results.

(1) The magnetic energy in the compression region increases only in strongly compressed materials $(n > 2)$ but the magnetic energy density and the field increase always.

(2) The regime of shock-wave convergence, the limiting field, and the final dimensions of the field compression region depend on the compression of the material and the parameter $\varepsilon = 8\pi w_K/B_0^2$, which is equal to the ratio of the kinetic energy density of the liner producing the shock wave w_K to the magnetic energy density in the compression region at the moment of start. In this case, the shock wave is always reflected from the field if the compressibility of the material is sufficiently high: $n > 2$. In more rigid materials $(n < 2)$, shock-wave reflection from the field occurs with low kinetic energy: $\varepsilon < (u/D)/(1 - 2u/D)$. In these cases, the generated magnetic field is finite. With increase in the above-mentioned limiting energy in rigid materials, the shock wave is incident on the axis, resulting in a theoretically unlimited increase in the field. This adds a new example of unlimited cumulation to the well-known list of such phenomena. Deceleration of rigid materials with low packing $(n \leqslant 2)$ by a field does not lead to a marked removal of energy from the wave: the magnetic energy in the compression region is lower than the initial energy, and the case of full convergence of waves with an infinite field on the axis does not contradict the energy conservation law. At this moment, the magnetic flux completely enters into the conductor, but the magnetic energy density has a theoretical tendency toward an unbounded increase, i.e., such generators are not appropriate for conversion of energy but are quite suitable for magnetic-field amplification.

(3) In a shock-wave generator, the field rise rate is lower than that in magnetic compression by an ideal liner but because of removal of the flux from the compression region and the corresponding decrease in the wave-reflection radius, the field amplification surpasses the classical limit (1).

In shock-wave compression of a magnetic field:

— Explosive evaporation of the conductor is eliminated because of the large thickness of the current layer, the presence of a gas-dynamic pressure jump, and continuous renewal of the material at the wave front;

— The stability of compressions increases. In our experiments, the cross-sectional dimension of the compressed region changed by more than a factor of 40 and was limited only by the accuracy of shock-wave convergence to the system axis. Theoretical studies of the stability of a plane shock-wave compression scheme performed jointly by researchers from the Institute of Physical Problems of the Russian Academy of Sciences (Troitsk), TRINITI, and the Defense Center of Sweden showed complete stabilization of the most critical short-wave disturbances whose length is lower than a certain critical value close to the dimensions of the compression region, and a considerable decrease in the growth rate of long-wave disturbances [26].

Some New Effects. Although considerable time elapsed since the publication of the idea and description of the first experiments, the shock-wave compression of a magnetic field has been studied inadequately. This is due to the fact that for well-known reasons, Russian researchers are currently deprived of the possibility of conducting serious experiments, and for Japanese scientists, investigation into the area of magnetic cumulation has been an unusual and rare episode, far from their scientific predilections and traditions. Nevertheless, some qualitatively new aspects typical of the shock-wave method of magnetic compression have been revealed and studied. The most interesting of them are as follows:

1. Calculations revealed oscillations of the wave and mass velocities of a material under certain conditions of shock-wave compression of a field.

2. Calculations yielded high electric fields.

3. It was found that the efficiency of shock-wave compression of a magnetic field depends strongly on the capability of a shock wave to trap a certain current in its front and to carry it with itself.

4. Investigation of the electromagnetic radiation flows accompanying experiments on shock-wave compression of a magnetic field.

The most important results pertaining to each of the four indicated items are briefly described below.

1. The first estimates of the possibilities of shock-wave magnetic field compression were obtained analytically for an ideally packed medium. In this model, the material behind the wave front was considered incompressible, and the packing parameter was considered constant. The following step involved allowance for the compressibility of the material and estimation of its influence on results of compression. Numerical modeling schemes were developed by Nagayama [27] and Barmin with collaborators [28]. Nagayama performed calculations using the Mie–Grüneisen approximation, and Barmin used approximations for the equation of state based on the theory of a free volume and, in a number of calculations, the model equation of state fitted to empirical data. Somewhat later, we also performed a similar work [29], where the Oh and Person approximation was employed to describe highly porous materials [30], and shock-adiabat parameters were extrapolated not for pressure, as in the Mie–Grüneisen model, but for the specific volume.

The most unexpected result from Barmin's calculations and ours was the finding of oscillations of the shock-front velocity and the mass velocity of material in a certain compression phase. Our results for generators with aluminum powder are presented in Fig. 2. Calculated distributions of the pressure, mass velocity, and flux of the material provided an understanding of details of the flow with oscillations of the wave velocity. Results of pressure calculations are presented in Fig. 3. At the beginning of compression, the magnetic pressure is insignificant, and with time, it increases monotonically. The pressure jump at the shock front has an approximately constant magnitude and is clearly seen against the background of the magnetic-pressure component. Initially, the total pressure behind the jump is almost constant. With time, it exhibits a downward tendency with recession from the shock wave to the material. This is due to the distribution of the magnetic field "frozen" into the material and the total pressure component produced by the field.

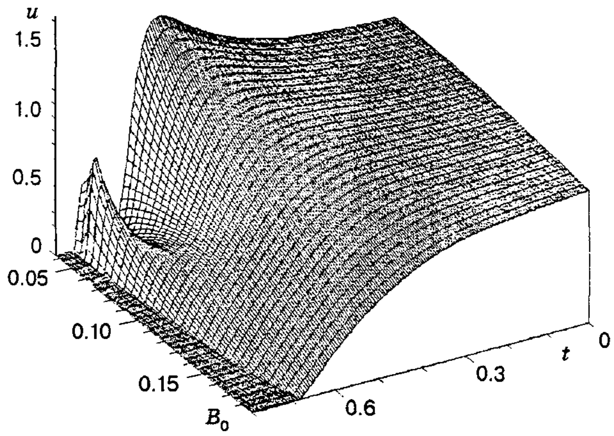

Fig. 2. Oscillations of the mass velocity at the shock-wave front in a shock-wave generator with aluminum powder.

In the compression phase near the minimum velocity, the magnetic field reaches a considerable magnitude and markedly decelerates the material. As a result, the wave velocity drops, the magnetic field is stabilized, and the pressure jump at the front decreases and is almost invisible against the background of the high magnetic pressure. In the depth of the compressed material, motion to the axis continues, as a result of which a high-pressure region (a clearly seen pressure bulge in Fig. 3) is formed there. Under the action of this pressure, the material on the periphery is decelerated, and in the axial region, it is accelerated — there is a redistribution of kinetic energy in the material, which is known as the phenomenon of hydrodynamic cumulation. The most nontrivial point in the calculations performed is that they revealed cumulation of kinetic energy in a strongly compressed material. The indicated motion features and their manifestation depend greatly on the choice of material and are observed in a narrow range of magnitudes of the initial magnetic field.

2. Numerical simulations of shock-wave compression of a magnetic field have been extensively performed at the National Defense Center of Sweden. Perhaps, all currently available equations of state and interpolations for conductivity of CsI were used in these calculations [31]. The calculations gave extremely high parameters for the state that arises at the end of compression and the fields thus produced (magnetic field 14 MG, electric field 200 kV/cm, current density 20 GA/cm^2, mass velocity 10 km/sec, hydrodynamic pressure more than 25 Mbar, energy density 210 kJ/g, density 30 g/cm^3, and temperature 10^6 K). The calculated parameters are impressive. And if something similar to the calculations occur in reality, it is possible to speak of the enormous breakthrough into a new area of extreme states of materials. The main problem is to obtain experimental evidence for this. Here again, one might expect interesting news: in a private communication from the U.S.A received by the author in 1999, it was stated that researchers from the Cavendish Laboratory had recorded very strong electric fields in shock-wave compression of a magnetic field in cylindrical samples made of a CsI-type material.

3. In our recent work, we considered a cycle of new problems of studying the spatial flow pattern of materials and the associated electromagnetic fields, magnitudes, and distributions of currents and electromagnetic energy flux under shock-wave compression in magnetic fields of condensed conducting materials and materials with phase changes "conductor–dielectric" and "dielectric–conductor" [32–35]. In these studies, we developed a closed model of electrodynamic processes accompanying shock compression of a material and established that, under certain conditions, a shock wave can trap a certain current and carry it within its front. The value of this current is determined by the compressibility and conductivity of the material and the thickness of the wave front. In addition, for the "dielectric–conductor" transition, it is determined by the position of the point of phase transition to the conducting state in the wave front. Such waves are called current-carrying waves.

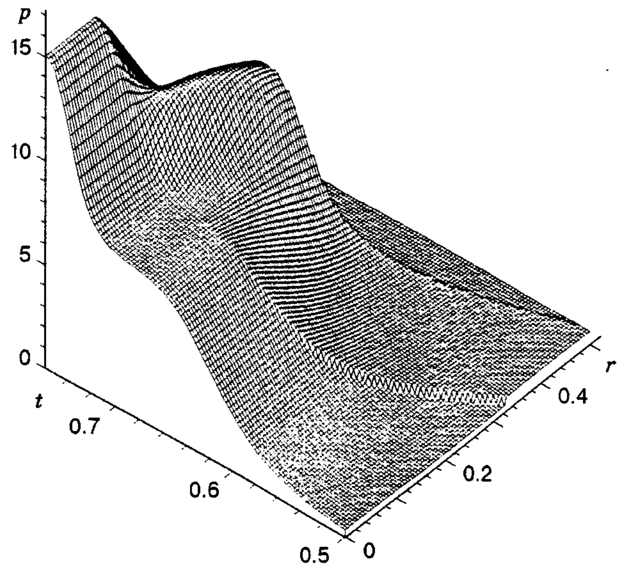

Fig. 3. Distribution of the total pressure.

It turned out that the presence of a current in a wave screens the flow of electromagnetic radiation from the shock-wave front and reduces the efficiency of magnetic compression, and the propagation of current-carrying waves is accompanied by a number of new electrodynamic effects. Thus, the entry of this wave into a conductor is accompanied by appearance of a "current–anticurrent" pair, which consists of two currents having the same magnitude and opposite directions, closed on each other, and located originally on the surface conductor surface. After passage of the wave, this virtual pair is decomposed into two substantial currents. The current in this case is separated from the anticurrent and carried away by the wave. The anticurrent remains on the surface of the conductor and diffuses from it into the conducting material. The same splitting of "current–anticurrent" pairs accompanies the decay of a shock wave on an interface between conducting materials with different shock properties. In this case, pairs "current–anticurrent" arise on the surfaces of both materials, i.e., in the case of a current-carrying wave, the classical decay of a hydrodynamic discontinuity on the interface is accompanied by occurrence of four currents. Apparently, in this case, corresponding magnetic dipole moments are produced, and in a number of cases, the propagation of current-carrying waves can be accompanied by occurrence of powerful flows of electromagnetic radiation. The radiation accompanying shock-wave compression of a magnetic field should be considered specially.

4. In 1993–1997, in various issues, including reviews of the modern state of the art of weapons, many publications appeared which reported that in magnetic cumulation experiments, it was possible to record microwave radiation power fluxes [36]. This aroused keen interest, primarily among the people responsible for the state and level of the national defense potential. Naturally, the Americans were ahead of the others. But physicists were also intrigued: if the reports are true, this opens up a new direction of energy cumulation research. The U.S. Defense Department organized a very strong team, financed it, and carried out, together with the authors of the publications, a very careful verification of the reports. Results were submitted at the VIIIth Megagauss Conference at Tallahassee (Florida) in October, 1998 [37]. The main result of the control tests performed was that microwave radiation was not revealed.

Despite the unfavorable results of the control tests, the question of the application of shock-wave compression of a magnetic field to production of powerful flows of electromagnetic radiation deserves special consideration. There is no doubt that, as a result of magnetic cumulation, fields of several Megagauss are generated in a material and a great amount of energy is accumulated. This energy should be removed somewhere after termination of compression. If the material remains in the conducting state, the reserve of magnetic energy is converted primarily to heat and partly to the mechanical motion of the conductor.

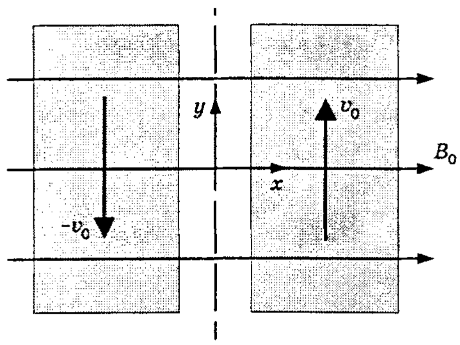

Fig. 4. Generation of a field by shear motion of a conductor.

However, it is difficult to expect that after convergence of the shock wave to the axis and the subsequent scattering, the compressed sample material remains continuous and conducting. Explosive units can be designed in such a way that even in the convergence stage, a shock wave in a material is followed by a rarefaction wave, which leads to loss of conductivity. In this case, occurrence of electromagnetic radiation is inevitable. Besides this simple scheme leading to production of an electromagnetic radiation source based on magnetic cumulation, there may be other methods for generation of the radiation by appropriate modulation in time of the magnetic moments produced by currents that arise under conditions of shock-wave compression. Some of the above problems were covered in [34] and [35] (in press).

Magnetic Cumulation in Hydrodynamic Flows of a Conductor.

Generation of Fields by Shear Flow of a Conducting Liquid.

Formulation of the Problem. Let us consider a plane slot between two infinite conductors whose surfaces are parallel and spaced $2a$ apart (Fig. 4). The conductivity of the material of the slot walls σ is assumed to be constant. At the initial moment, the magnetic field B_0 in the material is uniform and perpendicular to the boundaries of the slot. Let the walls be instantaneously set in motion at constant velocity v_0 in opposite directions perpendicular to the initial field. The coordinate axis x is directed along the field B_0, and the y axis is directed along the velocity. In view of the symmetry, we can consider half of the slot, by placing a perfectly conducting plane in the middle of the slot, and reverse the motion by moving over to a reference system attached to the conductor. In this case, an electric field of strength $(v_0/c)B_0$ directed along the z axis arises in unit length of the perfect conductor. The field produces a current that circulates around the slot. This current will generate a magnetic field B directed along the y axis.

Equation. The Faraday law of electromagnetic induction for the circuit superimposed on the induced current circulating over the surface of conductors leads to the equation

$$\frac{v_0}{c} B_0 - \frac{a}{c} \frac{dB}{dt} = -\left(\frac{j}{\sigma}\right)\Big|_{x=0}, \tag{4}$$

where j is the current density and σ is the conductivity on the surface of the conductor. In turn, the current density is related to its associated field by Ampere's equation, which, for the problem considered, leads to

$$j = \frac{c}{4\pi} \frac{\partial B}{\partial x},$$

or, after substitution into (4),

$$\frac{dB}{dt} = \left(\frac{c^2}{4\pi\sigma a} \frac{\partial B}{\partial x}\right)\Big|_{x=0} + \frac{v_0}{a} B_0. \tag{5}$$

Choosing B_0 as the scale of the magnetic field, the slot diameter a as the linear scale, and the time during which the conductor travels the slot diameter a/v_0 as the time scale, we bring Eq. (5) to dimensionless form

$$\frac{dB}{dt} = \left(\frac{1}{\mathrm{Re}_m} \frac{\partial B}{\partial x}\right)\Big|_{x=0} + 1, \tag{6}$$

in which the only determining parameter of the problem is the magnetic Reynolds number Re_m (2).

Solution of the Problem. In Eq. (6), the derivative of the field with respect to the normal to the boundary of the conductor can be determined from the magnitude of the magnetic field on the boundary using the equation of diffusion of the magnetic field into a plane conductor. Thus, we obtain the important relation [11]

$$\left(\frac{\partial B}{\partial x}\right)\Big|_{x=0} = -\sqrt{\frac{\mathrm{Re}_m}{\pi}}\,\frac{d}{dt}\int\limits_0^t \frac{B(\tau)}{\sqrt{t-\tau}}\,d\tau.$$

Substituting the latter into (6) and integrating it with respect to time, we arrive at the integral equation

$$B(t) = t - \sqrt{\frac{1}{\pi\,\mathrm{Re}_m}}\int\limits_0^t \frac{B(\tau)}{\sqrt{t-\tau}}\,d\tau,$$

which includes the only unknown function — the magnetic field in the slot $B(t)$. This equation can be solved by methods similar to the well-known method of solving Abel's equation [38]. The result is the following differential equation for $B(t)$:

$$\frac{dB}{dt} - \frac{B}{\mathrm{Re}_m} = 1 - \frac{2\sqrt{t}}{\sqrt{\pi\,\mathrm{Re}_m}}.$$

The solution of this equation subject to the initial condition $B(0) = 0$ has the form

$$B(t) = \mathrm{Re}_m(2\sqrt{t_r}/\sqrt{\pi} - 1 + B_*(t_r)), \quad B_*(t_r) = (1 - \Phi(\sqrt{t_r}))\exp(t_r), \quad t_r = t/\mathrm{Re}_m. \tag{7}$$

Here $\Phi(z)$ is the probability integral.

Analysis of the Solution. The most important features of the solution obtained are as follows.

1. In the solution there was a renormalization of time: the time scale is now not the time of displacement of the conductor at a distance equal to the slot diameter but a quantity that is larger by a factor of Re_m. According to the definition (2), this is the time of relaxation of the field from the plane slot, which is more adequate to the physical sense of the problem. The new time scale is denoted by t_r.

2. The field amplification coefficient is proportional to the magnetic Reynolds number multiplied by a function of time. This function is written in asymptotic form as

$$\frac{B(t)}{\mathrm{Re}_m} \approx t_r - \frac{4t_r^{3/2}}{3\sqrt{\pi}} + \frac{t_r^2}{2} - \frac{8t_r^{5/2}}{15\sqrt{\pi}} + \frac{t_r^3}{6}, \ t_r \ll 1, \quad \frac{B(t)}{\mathrm{Re}_m} \approx \frac{2}{\sqrt{\pi}}\sqrt{t_r} - 1 + \frac{1}{\sqrt{\pi t_r}}, \ t_r \to \infty. \tag{8}$$

The most remarkable point in the solution obtained is that the field amplification coefficient increases with time without limit in proportion to $\mathrm{Re}_m\sqrt{t_r}$, i.e., the solution obtained is one more example of a number of phenomena with unlimited rise in energy density.

It is not difficult to explain the result obtained: the force lines are stretched by the conductor along the slot in proportion to t_r and diffuse into the conductor to the depth of the skin layer, i.e., in proportion to $\sqrt{t_r}$, as a result of which the field amplification is proportional to $\sqrt{t_r}$.

The above asymptotic relations show a fundamentally new feature of magnetic-field generation in shear motion of a conductor compared to classical magnetic cumulation. It turns out that a specified amplification of the field $B(t) = \beta$ in shear motion can be obtained by two alternative methods. It is possible to work with large magnetic Reynolds numbers and to achieve a prescribed magnitude of the field over a short time $t_r \approx \beta/\mathrm{Re}_m$. In this case, the situation is nearly classical. But it is also possible to operate with small magnetic Reynolds numbers, even smaller than 1 — an unprecedented situation for classical magnetic cumulation. As follows from (8), in this case, it is possible to attain the desired field amplification in systems with very moderate rates of motion of conductors by ensuring a long time of generator operation $t_r \approx \pi\beta^2/(4\,\mathrm{Re}_m^2)$. Obviously, the displacements of the conductors, their dimensions and weight, and the overall dimensions of the facilities used may be cyclopic.

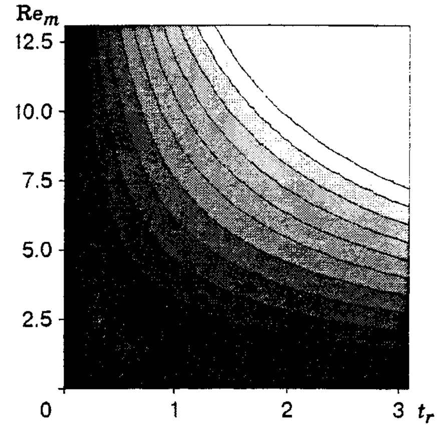

Fig. 5. Level lines of magnetic field amplification from 1 to 10.

The above conclusions are illustrated by the numerical results presented in Fig. 5, which shows level lines of the function $B(t)$ plotted with a step of 1 in the coordinates (t_r, Re_m) for field amplification from 1 to 10. It not easy to see the above-mentioned feature of magnetic-field amplification in the problem considered. From the given plots it also follows that to obtain moderate field amplification coefficients β, it is most reasonable to use rather moderate values of both the time of pumping of the cavity and magnetic Reynolds numbers.

Since at the beginning of operation of the current generator considered there is no cavity in the walls, the "going up" of this type of strong magnetic field sources proceeds rather slowly and a considerable time is required to produce a high current that leads to effective removal of mechanical energy from the moving conductor.

Some Estimates. To estimate the orders of magnitude of the quantities, in conclusion we give some figures that characterize the problem considered. We consider a slot with a diameter of 1 cm in a copper conductor. Assuming that the magnetic viscosity of copper is equal to 160 cm^2/sec [39], for the time of relaxation from the slot considered, we obtain $\tau_r = 1/160$ sec. Let the field be generated within time $t_r = 10$. For the chosen relaxation time, this is 1/16 sec. In this time, the skin layer increases to about $3a$, i.e., a conductor with a thickness of 10 cm can be considered thick in order that formulas obtained can be used to estimate the possible results in order of magnitude. From (7) it follows that, in this case, $B(10) = 2.74\,\mathrm{Re}_m$, i.e., for $\mathrm{Re}_m = 1$ and an initial field of 200 kG, one can hope to achieve 0.5 MG. The value $\mathrm{Re}_m = 1$ corresponds to a very moderate rate of motion of the conductor, only 160 cm/sec, i.e., the time of field generation corresponds to a quite realistic displacement of the conductor of about 10 cm.

At present, tremendous efforts are made to design hybrid magnets for fields of 300–500 kG. The estimates obtained give hope that competitive mechanical machines capable of generating the same field can be designed using the principle of stretching of the force lines of magnetic fields "frozen" in a conductor. Certainly, such machines will be similar in appearance to a sort of a steam locomotive but they can be quite suitable for physical studies. After all, P. L. Kapitsa opened the region of strong magnetic fields also using a huge mechanical generator.

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